Quantum holographic teleportation of light fields

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We describe a continuous variable teleportation scheme that allows to teleport with high fidelity the quantum state of broadband multimode electromagnetic field. We call this scheme "quantum holographic teleportation" because it allows for reconstruction of an optical wavefront preserving its quantum correlations in space-time.

Quantum teleportation allows for transportation of an arbitrary quantum state of a field from one place to another using classical information exchange. Initially proposed for discrete variables [1], later on teleportation was extended to continuous-variable schemes [2,3]. Experimental demonstration of quantum teleportation for discrete variables was realized in [4,5] for single-photon polarization states, and continuous-variable teleportation in [6] for coherent state of electromagnetic field. Next challenging task is teleportation of truly nonclassical states like entangled states or so-called "entanglement swapping". The concept of entanglement swapping was initially introduced for single-photon polarization states [7] and has already been realized experimentally for single photons [8]. There are several proposals of entanglement swapping for continuous-variable teleportation schemes [9–11]. Apart from obvious interest to teleportation due to its fundamental quantum nature nonexistent in classical physics, there is a practical interest to this phenomenon stirred by potential applications in quantum error correction [12–14], quantum dence coding [15], and quantum cryptography [16].

To date, most theoretical schemes consider quantum teleportation of just a single-mode state of the field. Such an assumption greatly simplifies analysis of the teleportation protocol and calculation of parameters describing the performance of the scheme. However, in reality one has to deal with optical signals distributed in space-time which have characteristic spatio-temporal scales such as coherence time and coherence area, for example. To understand the role of these parameters in teleportation process we have to abandon the single-mode approximation and generalize quantum teleportation for multimode states of electromagnetic field.

While broadband teleportation of time-dependent signals has been already discussed in the literature [17], the spatial aspects of the problem has been ignored so far. In this paper we propose a full spatio-temporal teleportation protocol. Our scheme allows us to teleport the quantum state of the distributed in space-time electromagnetic field, which can carry a spatial information like an optical image, or spatio-temporal information like an animation or a movie.

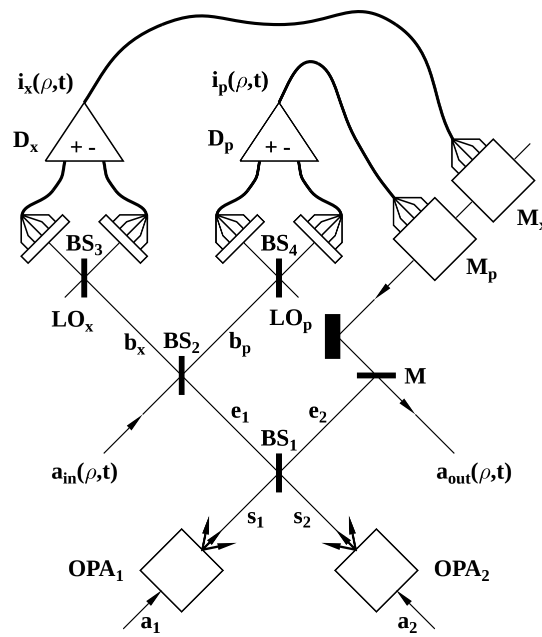

FIG. 1. Schematic of holographic teleportation...

Our teleportation scheme is similar to that described in [3] and is shown in the figure. The input light field to be teleported from Alice to Bob is denoted by $A_{\rm in}(\vec{\rho},t)$, where $\vec{\rho} = (x,y)$ is the transverse coordinate. Two quadrature components of the light field are detected "point-by-point" by two balanced homodyne detectors D_x and D_p formed by high efficiency multipixel photodetection matrices (CCD). The spatio-temporal quantum fluctuations of these quadrature components are locally imprinted into the photocurrents $i_x(\vec{\rho},t)$ and $i_p(\vec{\rho},t)$ on the output of individual pixels of CCD cameras. These photocurrents are sent from Alice to Bob via two multichannel parallel classical communication lines. Bob uses these photocurrents for reconstruction of the field $A_{\rm out}(\vec{\rho},t)$ via two multichannel modulators M_x and M_p which modulate in space and time an incoming plane coherent light wave.

The essential part of the teleportation scheme is a pair of traveling-wave optical parametric amplifiers OPA_1 and OPA_2 used for creation of two broadband multimode entangled Einstein-Podolsky-Rosen (EPR) light beams. Owing to the multimode nature of entanglement created by the OPAs, our scheme allows for parallel teleportation with optimum fidelity of N elements of the input wavefront, preserving the space-time correlations between these elements. This number is given by the ratio of the wavefront cross-section to the coherence area of the light created by OPAs. In the generic teleportation scheme [3] $N = 1$.

The EPR beams $E_n(\vec{\rho},t)$, n = 1,2, are created by the interference mixing at the 50:50 beam splitter BS_1 of the fields $S_m(\vec{\rho},t)$, m = 1,2, in broadband multimode squeezed state,

$$E_n(\vec{\rho},t) = \sum_{m=1,2} R_{nm} S_m(\vec{\rho},t), \tag{1}$$

where

$$\{R_{nm}\} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \tag{2}$$

is the scattering matrix of the beam splitter BS_1. The light waves $S_m(\vec{\rho},t)$ in the broadband multimode squeezed state are created by two traveling-wave optical parametric amplifiers OPA_1 and OPA_2. The detailed description of properties of such squeezed light can be found, for example in Ref. [18]. The transformation of the input fields $A_m(\vec{\rho},t)$ in the vacuum state into the output fields $S_m(\vec{\rho},t)$ in the broadband multimode squeezed state is described in terms of the Fourier components of these operators in frequency and spatial-frequency domain,

$$s_m(\vec{q},\Omega) = \int d\vec{\rho}\, dt \exp[i(\Omega t - \vec{q}\cdot\vec{\rho})] S_m(\vec{\rho},t), \tag{3}$$

and similar for $a_m(\vec{q},\Omega)$. These Fourier components are related as follows,

$$s_m(\vec{q},\Omega) = U_m(\vec{q},\Omega) a_m(\vec{q},\Omega) + V_m(\vec{q},\Omega) a_m^\dagger(-\vec{q},-\Omega), \tag{4}$$

where the coefficients $U_m(\vec{q},\Omega)$ and $V_m(\vec{q},\Omega)$ depend on the pump-field amplitudes of the OPAs, their nonlinear susceptibilities and the phase-matching conditions [18].

The spatial and temporal scales of our teleportation scheme are determined by the orientation angle $\psi_m(\vec{q},\Omega)$ of the major axis of the squeezing ellipse,

$$\psi_m(\vec{q},\Omega) = \frac{1}{2} \arg\{U_m(\vec{q},\Omega) V_m(-\vec{q},-\Omega)\}, \tag{5}$$

and by the degree of squeezing $r_m(\vec{q},\Omega)$,

$$e^{\pm r_m(\vec{q},\Omega)} = |U_m(\vec{q},\Omega)| \pm |V_m(\vec{q},\Omega)|. \tag{6}$$

In analogy to the single-mode EPR beams, the multimode EPR beams are created if squeezing in both channels is effective, and the squeezing ellipses are oriented in orthogonal directions. For simplicity we shall assume that OPA_1 and OPA_2 have such properties that,

$$\begin{aligned} U_1(\vec{q},\Omega) &= U_2(\vec{q},\Omega) \equiv U(\vec{q},\Omega), \\ V_1(\vec{q},\Omega) &= -V_2(\vec{q},\Omega) \equiv V(\vec{q},\Omega). \end{aligned} \tag{7}$$

This situation is realized, for example, for broadband multimode squeezing produced by an OPA with type-II phase matching [19]. In the latter case two independent squeezed beams correspond to two orthogonal polarization components of the field with the following properties,

$$\begin{aligned} r_1(\vec{q},\Omega) &= r_2(\vec{q},\Omega) \equiv r(\vec{q},\Omega), \\ \psi_1(\vec{q},\Omega) &= \psi_2(\vec{q},\Omega) \pm \pi/2 \equiv \psi(\vec{q},\Omega). \end{aligned} \tag{8}$$

Under these conditions the EPR fields are entangled for the frequencies Ω and spatial frequencies $\vec{q}$ within the phase matching of the OPA.

For observation of two quadrature components of the input field $A_{\rm in}(\vec{\rho},t)$ the input beam is mixed with one of two EPR beams, $E_1(\vec{\rho},t)$, at the beam splitter BS_2 with the same scattering matrix as in Eq. (2). This gives the input fields of the balanced homodyne detectors D_x, D_p as

$$\begin{aligned} B_x(\vec{\rho},t) &= \frac{1}{\sqrt{2}}\big(A_{\rm in}(\vec{\rho},t) + E_1(\vec{\rho},t)\big), \\ B_p(\vec{\rho},t) &= \frac{1}{\sqrt{2}}\big(-A_{\rm in}(\vec{\rho},t) + E_1(\vec{\rho},t)\big). \end{aligned} \tag{9}$$

These fields in turn are mixed with the local oscillator fields LO_x and LO_p having complex amplitudes B_0 and $-iB_0$, where B_0 is real, at beam splitters BS_3 and BS_4 with the same scattering matrices as for BS_1 and BS_2. We shall assume that pixels of the CCD matrices have the area much smaller than the coherence area S_c of the EPR beams. In this case it can be shown that the photocurrents collected from individual pixels of D_x and D_p at the point $\vec{\rho}$ are given by,

$$\begin{aligned} i_x(\vec{\rho},t) &= B_0\big(B_x(\vec{\rho},t) + B_x^\dagger(\vec{\rho},t)\big), \\ i_p(\vec{\rho},t) &= B_0\frac{1}{i}\big(B_p(\vec{\rho},t) - B_p^\dagger(\vec{\rho},t)\big). \end{aligned} \tag{10}$$

These photocurrents are sent from Alice to Bob via two multichannel classical communication lines and are used by Bob for local modulation of an external coherent wave, phase matched with the squeezed fields [6,9]. In the modulated beam the field component $\sim i_x(\vec{\rho},t) - i i_p(\vec{\rho},t)$ is created. The teleported field $A_{\rm out}(\vec{\rho},t)$ is obtained by interference mixing on the mirror M with hight reflectivity of the modulated field with the second EPR beam $E_2(\vec{\rho},t)$,

$$A_{\rm out}(\vec\rho,t) = E_2(\vec\rho,t) + g\big(i_x(\vec\rho,t) - i i_p(\vec\rho,t)\big). \qquad (11)$$

Here g is the coupling constant which takes into account the efficiency of modulation and the transmission of the mirror M. The teleportation takes place when $gB_0\sqrt{2} = 1$. We find the teleported field $A_{\rm out}(\vec\rho,t)$ in the form,

$$A_{\rm out}(\vec\rho,t) = A_{\rm in}(\vec\rho,t) + F(\vec\rho,t), \qquad (12)$$

where

$$F(\vec\rho,t) = E_2(\vec\rho,t) + E_1^\dagger(\vec\rho,t), \qquad (13)$$

is the noise field, added by the teleportation process. In the ideal case of perfect entanglement of two EPR beams at all frequencies Ω and spatial frequencies $\vec q$ the terms $E_2(\vec\rho,t)$ and $E_1^\dagger(\vec\rho,t)$ are perfectly anticorrelated and their quantum fluctuation cancel each other. This would correspond to the perfect "point-to-point" in space and instantaneous in time teleportation of the quantum state of the input field with an arbitrary distribution in space and time, $A_{\rm out}(\vec\rho,t) = A_{\rm in}(\vec\rho,t)$. However such teleportation would require infinitely large energy of EPR beams. Indeed, firstly as in the single-mode case, one would have to achieve an infinite squeezing per single coherence volume of an EPR beam. Additionally, since now we have broadband multimode entanglement, one would need an infinite number of elementary coherence volumes in the EPR beams. In practice teleportation will never be point-to-point in space and instantaneous in time but always "on average" within some spatial area and within some finite time interval.

In order to verify that teleportation has actually taken place and to evaluate its quality we shall follow the strategy employed in Ref. [6] and introduce the third party, Victor. He will make two measurements with the fields $A_{\rm in}(\vec\rho,t)$ and $A_{\rm out}(\vec\rho,t)$ and according to the results of these measurements he will decide whether teleportation was successful or not.

We shall assume that Victor performs his own homodyne detection measurement of an arbitrary quadrature component of the *in* and *out* fields determined by the angle ϕ,

$$i^{(\phi)}_{\rm out}(\vec\rho,t) = A_0\big(A_{\rm out}(\vec\rho,t)e^{-i\phi} + A^\dagger_{\rm out}(\vec\rho,t)e^{i\phi}\big), \qquad (14)$$

where A_0 is the real amplitude of Victor's local oscillator. For evaluation of the teleportation quality Victor observes the photocurrent noise spectrum of the teleported field defined as,

$$\big(\delta i^2_{\rm out}\big)^{(\phi)}_{\vec q,\Omega} = \int d\vec\rho\, dt \exp[i(\Omega t - \vec q\cdot\vec\rho)]$$
$$\times\, \langle \delta i^{(\phi)}_{\rm out}(\vec 0,0)\delta i^{(\phi)}_{\rm out}(\vec\rho,t)\rangle, \qquad (15)$$

where $\delta i^{(\phi)}_{\rm out}(\vec\rho,t)$ is the fluctuation of the photocurrent around its mean value. He compares this spectrum with an analogous noise spectrum $\delta i^{(\phi)}_{\rm in}(\vec\rho,t)$ for the input field. Using Eq. (12) and relations between the EPR fields and the input fields of the OPAs we obtain,

$$\big(\delta i^2_{\rm out}\big)^{(\phi)}_{\vec q,\Omega} = \big(\delta i^2_{\rm in}\big)^{(\phi)}_{\vec q,\Omega} + 2A_0^2\Big\{e^{-2r(\vec q,\Omega)}\cos^2\psi(\vec q,\Omega)$$
$$+ e^{2r(\vec q,\Omega)}\sin^2\psi(\vec q,\Omega)\Big\}. \qquad (16)$$

Let us consider first teleportation of a classical plane monochromatic wave, i. e. when the Fourier component of $A_{\rm in}(\vec\rho,t)$ with $\vec q = 0$ and $\Omega = 0$ is in coherent state and all other components in vacuum state. In this case $\big(\delta i^2_{\rm in}\big)^{(\phi)}_{\vec q,\Omega} = a_0^2$. Without squeezing, $r(\vec q,\Omega) = 0$, we obtain $\big(\delta i^2_{\rm out}\big)^{(\phi)}_{\vec q,\Omega} = 3\big(\delta i^2_{\rm in}\big)^{(\phi)}_{\vec q,\Omega}$. In this classical limit of teleportation [6] the three-dimensional noise spectrum of photocurrent is multiplied by the same factor 3, as the one-dimensional Ω-dependent spectrum in the single-mode case.

Quantum teleportation superseding this classical limit is possible when (i) there is an effective squeezing, $r(\vec q,\Omega) \gg 1$, in a certain region of frequencies $\vec q$ and Ω, and (ii) the orientation angle $\psi(\vec q,\Omega)$ within this region is $\psi(\vec q,\Omega) \simeq 0$.

It was shown earlier that the Ω-dependence of the angle $\psi(\vec q,\Omega)$ can be compensated by the frequency-dependent refraction index of nonlinear medium [20], and the $\vec q$-dependence is to large extent compensated by a thin lens properly inserted after the OPA [21,18]. Assuming that OPAs have phase matching degenerate in frequency and angle, we can introduce the coherence area $S_c = (2\pi/q_c)^2$ and the coherence time $T_c = 2\pi/\Omega_c$ for the EPR fields. The squeezing is effective for $|q| \le q_c/2$ and $|\Omega| \le \Omega_c/2$ The three-dimensional photocurrent noise spectra of the *in* and *out* fields become identical within this range of frequencies and spatial frequencies, $\big(\delta i^2_{\rm out}\big)^{(\phi)}_{\vec q,\Omega} = \big(\delta i^2_{\rm in}\big)^{(\phi)}_{\vec q,\Omega}$. This corresponds to quantum teleportation which cannot be achieved without broadband multimode EPR beams shared by Alice and Bob.

The low-frequency noise suppression in Eq. (16) means that if the photocurrents are collected from the area $S \gg S_c$ during the time $T \gg T_c$, the fluctuations in the *in* and *out* measurements have similar statistics. In the opposite case of measurement with $S \ll S_c$, $T \ll T_c$ the high-frequency noise contribution in Eq. (16) degrades the quality of teleportation. Therefore, for the space-time distributed fields the concept of teleportation fidelity must include coarse-grained description with the scales S_c, T_c of the EPR beams.

The teleportation fidelity is degraded due to the noise field $F(\vec\rho,t)$ in Eq. (12). The commutation relations for $F(\vec\rho,t)$ correspond to classical noise,

$$[F(\vec\rho,t), F^\dagger(\vec\rho\,',t')] = 0, \quad [F(\vec\rho,t), F(\vec\rho\,',t')] = 0. \qquad (17)$$

The statistics of this noise field are determined in the

most general form by the characteristic functional,

$$\mathcal{F}(\lambda,\lambda^\star)=\langle\exp\Big\{\int d\vec{\rho}\,dt\Big(\lambda(\vec{\rho},t)F^\star(\vec{\rho},t)$$
$$-\lambda^\star(\vec{\rho},t)F(\vec{\rho},t)\Big)\Big\}\rangle. \tag{18}$$

Our calculations which will be published elsewhere give,

$$\mathcal{F}(\lambda,\lambda^\star)=\exp\Big\{-\int d\vec{\rho}\,d\vec{\rho}\,'dt\,dt'\lambda(\vec{\rho},t)\lambda^\star(\vec{\rho}\,',t')$$
$$\times G(\vec{\rho}-\vec{\rho}\,',t-t')\Big\}, \tag{19}$$

where the Fourier transform of the Green function $G(\vec{\rho},t)$ reads,

$$G(\vec{q},\Omega)=|U(\vec{q},\Omega)-V^\star(-\vec{q},-\Omega)|^2$$
$$=e^{-2r(\vec{q},\Omega)}\cos^2\psi(\vec{q},\Omega)+e^{2r(\vec{q},\Omega)}\sin^2\psi(\vec{q},\Omega). \tag{20}$$

It follows from Eq. (19) that the noise is Gaussian and the correlation functions of the fields $F(\vec{\rho},t)$ and $F^\star(\vec{\rho},t)$ of an arbitrary order are expressed in standard way via the second-order correlation functions

$$\langle F(\vec{\rho},t)F^\star(\vec{\rho}\,',t')\rangle=G(\vec{\rho}-\vec{\rho}\,',t-t'). \tag{21}$$

Incidentally, similar Green function describes the photocurrent correlations in space-time by the homodyne detection of multimode squeezed light [22].

When squeezing and entanglement are not present, the Green function is δ-correlated in space-time,

$$G(\vec{\rho},t)=\delta(\vec{\rho})\delta(t). \tag{22}$$

In presence of an effective entanglement with the scales S_c, T_c the positive δ-correlated term in Eq. (22) is accompanied by a negative term due to spatio-temporal anticorrelations on the scales S_c, T_c. Consider the field variables, averaged over the pixel S_j of area S within the time window T_i of duration T:

$$F(j,i)=\frac{1}{\sqrt{ST}}\int_{S_j}d\vec{\rho}\int_{T_i}dtF(\vec{\rho},t). \tag{23}$$

The Green function for these averaged variables goes over to the covariance matrix $\langle F(j,i)F^\star(j',i')\rangle$. For effective squeezing, $r(\vec{q},\Omega)\gg 1$, and large sampling volume, $S\gg S_c$, $T\gg T_c$, we obtain

$$\langle F(j,i)F^\star(j',i')\rangle\to\delta_{jj'}\delta_{ii'}e^{-2r(\vec{0},0)}. \tag{24}$$

These properties of the noise correlation function mean that noise on the scales $S\ll S_c$, $T\ll T_c$ is not eliminated, but on the scales $S\geq S_c$, $T\geq T_c$ effective entanglement results in significant noise suppression.

To conclude, we have proposed the protocol for quantum teleportation of the distributed in space-time light fields. The protocol is based on the entanglement between the corresponding coherence volumes cT_cS_c of the broadband multimode EPR fields produced by two traveling-wave OPAs. Every such volume determines an elementary degree of freedom of the input field, whose quantum state can be effectively teleported, i. e. the "resolving power" of teleportation.

It follows from Eq. (12), which relates in the Heisenberg picture the *out* field to *in* field, that any average product of the *out* fields, taken at arbitrary space-time points, is equal to the analogous average product of the *in* fields plus average noise term which can be made small using entangled EPR beams. That is, our protocol preserves the space-time quantum correlations in the *in* field. Teleportation with these features can be called *quantum holographic teleportation*.

In fact, in our three-dimensional generalization of the continuous variable teleportation protocol [3,6] one can easily recognize an extension to the quantum domain of the conventional non-stationary holography. As in holography, the distributed in space-time input field is mixed with the local oscillator waves. Classical photocurrent densities $i_x(\vec{\rho},t)$ and $i_p(\vec{\rho},t)$ are equivalent to non-stationary holograms, each for one of two quadrature components. These holograms are transmitted via classical multichannel communication lines and used for reconstruction of both quadrature components of the input field by means of non-stationary modulation of the beam from an external laser. The novel feature, which converts the holography to the quantum holographic teleportation is a pair of broadband multimode EPR beams shared by Alice and Bob.

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